

Chapter 1 – Electric Circuit Variables

Exercises

Exercise 1.2-1 Find the charge that has entered an element by time t when $i = 8t^2 - 4t$ A, $t \geq 0$. Assume $q(t) = 0$ for $t < 0$.

Answer: $q(t) = \frac{8}{3}t^3 - 2t^2$ C

Solution:

$$i(t) = 8t^2 - 4t \text{ A}$$

$$q(t) = \int_0^t i d\tau + q(0) = \int_0^t (8\tau^2 - 4\tau) d\tau + 0 = \frac{8}{3}\tau^3 - 2\tau^2 \Big|_0^t = \frac{8}{3}t^3 - 2t^2 \text{ C}$$

Exercise 1.2-2 The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

Answer: $i(t) = \frac{d}{dt} 4 \sin 3t = 12 \cos 3t$ A

Solution:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt} 4 \sin 3t = 12 \cos 3t \text{ A}$$

Exercise 1.3-1 Which of the three currents, $i_1 = 45 \mu\text{A}$, $i_2 = 0.03 \text{ mA}$, and $i_3 = 25 \times 10^{-4} \text{ A}$, is largest?

Answer: i_3 is largest.

Solution:

$$i_1 = 45 \mu\text{A} = 45 \times 10^{-6} \text{ A} < i_2 = 0.03 \text{ mA} = .03 \times 10^{-3} \text{ A} = 3 \times 10^{-5} \text{ A} < i_3 = 25 \times 10^{-4} \text{ A}$$

Exercise 1.5-1 Figure E 1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

- (a) Which of the devices supply 12 W?
- (b) Which of the devices absorb 12 W?
- (c) What is the value of the power received by device *B*?
- (d) What is the value of the power delivered by device *B*?
- (e) What is the value of the power delivered by device *D*?

Answers: (a) *B* and *C*, (b) *A* and *D*, (c) -12 W, (d) 12 W, (e) -12 W

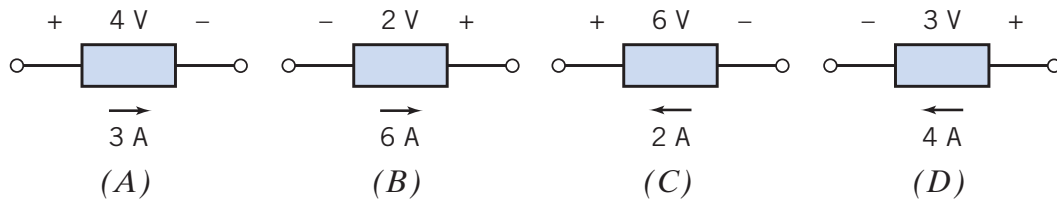


Figure E 1.5-1

Solution:

(a) *B* and *C*. The element voltage and current do not adhere to the passive convention in *B* and *C* so the product of the element voltage and current is the power supplied by these elements.

(b) *A* and *D*. The element voltage and current adhere to the passive convention in *A* and *D* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) -12 W. The element voltage and current do not adhere to the passive convention in *B*, so the product of the element voltage and current is the power received by this element: $(2 \text{ V})(6 \text{ A}) = -12 \text{ W}$. The power supplied by the element is the negative of the power delivered to the element, 12 W.

(d) 12 W

(e) -12 W. The element voltage and current adhere to the passive convention in *D*, so the product of the element voltage and current is the power received by this element: $(3 \text{ V})(4 \text{ A}) = 12 \text{ W}$. The power supplied by the element is the negative of the power received to the element, -12 W.

Problems

Section 1-2 Electric Circuits and Current Flow

P1.2.1 The total charge that has entered a circuit element is $q(t) = 1.25(1 - e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

Answer: $i(t) = 6.25e^{-5t}$ A

Solution:
$$i(t) = \frac{d}{dt} 1.25(1 - e^{-5t}) = \underline{6.25e^{-5t}} \text{ A}$$

P 1.2-2 The current in a circuit element is $i(t) = 4(1 - e^{-5t})$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^0 i(\tau) d\tau = \int_{-\infty}^0 0 d\tau = 0$

Answer: $q(t) = 4t + 0.8e^{-5t} - 0.8$ C for $t \geq 0$

Solution:

$$q(t) = \int_0^t i(\tau) d\tau + q(0) = \int_0^t 4(1 - e^{-5\tau}) d\tau + 0 = \int_0^t 4 d\tau - \int_0^t 4e^{-5\tau} d\tau = 4t + \underline{\frac{4}{5}e^{-5t} - \frac{4}{5}} \text{ C}$$

P 1.2-3 The current in a circuit element is $i(t) = 4 \sin 3t$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^0 i(\tau) d\tau = \int_{-\infty}^0 0 d\tau = 0$

Solution:

$$q(t) = \int_0^t i(\tau) d\tau + q(0) = \int_0^t 4 \sin 3\tau d\tau + 0 = -\frac{4}{3} \cos 3\tau \Big|_0^t = \underline{-\frac{4}{3} \cos 3t + \frac{4}{3}} \text{ C}$$

P 1.2-4 The current in a circuit element is $i(t) = \begin{cases} 0 & t < 2 \\ 2 & 2 < t < 4 \\ -1 & 4 < t < 8 \\ 0 & 8 < t \end{cases}$ where the units of current are A

and the units of time are s. Determine the total charge that has entered a circuit element for $t \geq 0$.

Answer:

$$q(t) = \begin{cases} 0 & t < 2 \\ 2t-4 & 2 < t < 4 \\ 8-t & 4 < t < 8 \\ 0 & 8 < t \end{cases} \text{ where the units of charge are C.}$$

Solution:

$$q(t) = \int_{-\infty}^t i(\tau) d\tau = \int_{-\infty}^t 0 d\tau = \underline{0 \text{ C}} \text{ for } t \leq 2 \text{ so } q(2) = 0.$$

$$q(t) = \int_2^t i(\tau) d\tau + q(2) = \int_2^t 2 d\tau = 2\tau \Big|_2^t = \underline{2t-4 \text{ C}} \text{ for } 2 \leq t \leq 4. \text{ In particular, } q(4) = 4 \text{ C.}$$

$$q(t) = \int_4^t i(\tau) d\tau + q(4) = \int_4^t -1 d\tau + 4 = -\tau \Big|_4^t + 4 = \underline{8-t \text{ C}} \text{ for } 4 \leq t \leq 8. \text{ In particular, } q(8) = 0 \text{ C.}$$

$$q(t) = \int_8^t i(\tau) d\tau + q(8) = \int_8^t 0 d\tau + 0 = \underline{0 \text{ C}} \text{ for } 8 \leq t.$$

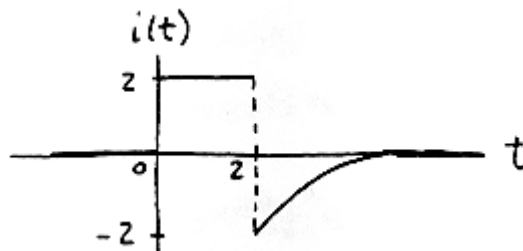
P 1.2-5 The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t \leq 2 \\ 3 + e^{-2t(t-2)} & t > 2 \end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \geq 0$.

Solution:

$$i(t) = \frac{dq(t)}{dt} \quad i(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 < t < 2 \\ -2e^{-2(t-2)} & t > 2 \end{cases}$$



P 1.2-6 An electroplating bath, as shown in Figure P 1.2-6, is used to plate silver uniformly onto objects such as kitchen ware and plates. A current of 600 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?

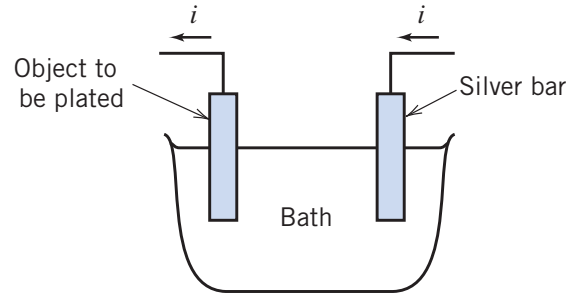


Figure P 1.2-6

Solution:

$$i = 450 \text{ A} = 450 \frac{\text{C}}{\text{s}}$$

$$\text{Silver deposited} = 450 \frac{\text{C}}{\text{s}} \times 20 \text{ min} \times 60 \frac{\text{s}}{\text{min}} \times 1.118 \frac{\text{mg}}{\text{C}} = 6.0372 \times 10^5 \text{ mg} = \underline{\underline{603.72 \text{ g}}}$$

P1.2-7 Find the charge $q(t)$ and sketch its waveform when the current entering a terminal of an element is as shown in Figure P1.2-7. Assume that $q(t) = 0$ for $t < 0$.

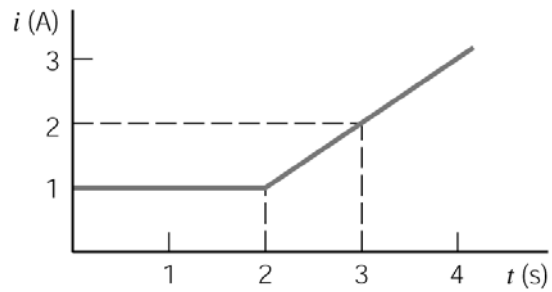


Figure P1.2-7

Solution:

$$i(t) = \begin{cases} 1 & \text{when } 0 < t \leq 2 \\ t-1 & \text{when } 2 \leq t \end{cases}$$

and

$$q(t) = \int_0^t i(\tau) d\tau + q(0) = \int_0^t i(\tau) d\tau$$

since $q(0) = 0$.

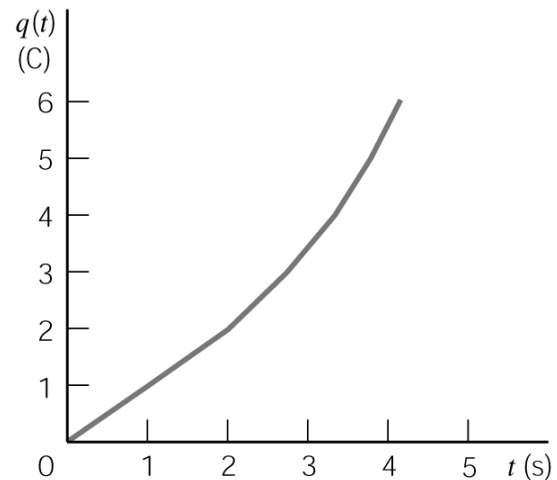
When $0 < t \leq 2$, we have

$$q = \int_0^t 1 d\tau = t \quad \text{C}$$

When $t \geq 2$, we have

$$\begin{aligned} q &= \int_0^t i(\tau) d\tau = \int_0^2 1 d\tau + \int_2^t (\tau-1) d\tau \\ &= \tau \Big|_0^2 + \frac{\tau^2}{2} \Big|_2^t - \tau \Big|_2^t = \frac{t^2}{2} - t + 2 \quad \text{C} \end{aligned}$$

The sketch of $q(t)$ is shown to the right..



Section 1-3 Systems of Units

P 1.3-1 A constant current of $3.2 \mu\text{A}$ flows through an element. What is the charge that has passed through the element in the first millisecond?

Answer: 3.2 nC

Solution:

$$\Delta q = i \Delta t = (3.2 \times 10^{-6} \text{ A})(1 \times 10^{-3} \text{ s}) = 3.2 \times 10^{-9} \text{ As} = 3.2 \times 10^{-9} \text{ C} = \underline{3.2 \times 10^{-9} \text{ nC}}$$

P 1.3-2 A charge of 45 nC passes through a circuit element during a particular interval of time that is 5 ms in duration. Determine the average current in this circuit element during that interval of time.

Answer: $i = 9 \mu\text{A}$

Solution:

$$i = \frac{\Delta q}{\Delta t} = \frac{45 \times 10^{-9}}{5 \times 10^{-3}} = 9 \times 10^{-6} = \underline{9 \mu\text{A}}$$

P 1.3-3 Ten billion electrons per second pass through a particular circuit element. What is the average current in that circuit element?

Answer: $i = 1.602 \text{ nA}$

Solution

$$\begin{aligned} i &= \left[10 \text{ billion } \frac{\text{electron}}{\text{s}} \right] \left[1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}} \right] = \left[10 \times 10^9 \frac{\text{electron}}{\text{s}} \right] \left[1.602 \times 10^{-19} \frac{\text{C}}{\text{electron}} \right] \\ &= 10^{10} \times 1.602 \times 10^{-19} \frac{\text{electron}}{\text{s}} \frac{\text{C}}{\text{electron}} \\ &= 1.602 \times 10^{-9} \frac{\text{C}}{\text{s}} = \underline{1.602 \text{ nA}} \end{aligned}$$

P1.3-4 The charge flowing in a wire is plotted in Figure P1.3-4. Sketch the corresponding current.

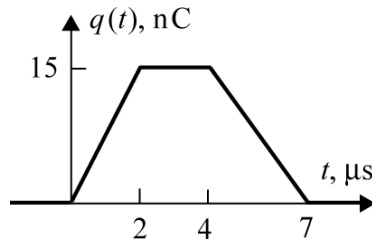
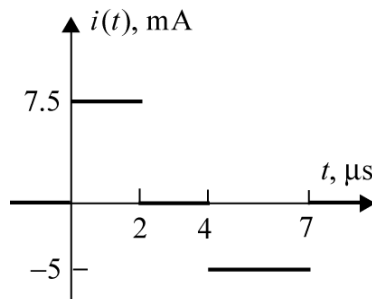


Figure P1.3-4

P1.3-4

$$i(t) = \frac{d}{dt}q(t) = \text{the slope of the } q \text{ versus } t \text{ plot} = \begin{cases} \frac{15 \times 10^{-9}}{2 \times 10^{-6}} = 7.5 \times 10^{-3} = 7.5 \text{ mA} & \text{when } 0 < t < 2 \mu\text{s} \\ \frac{15 \times 10^{-9}}{3 \times 10^{-6}} = -5 \times 10^{-3} = -5 \text{ mA} & \text{when } 4 \mu\text{s} < t < 7 \mu\text{s} \\ 0 & \text{otherwise} \end{cases}$$



P1.3-5 The current in a circuit element is plotted in Figure P1.3-5. Sketch the corresponding charge flowing through the element for $t > 0$.

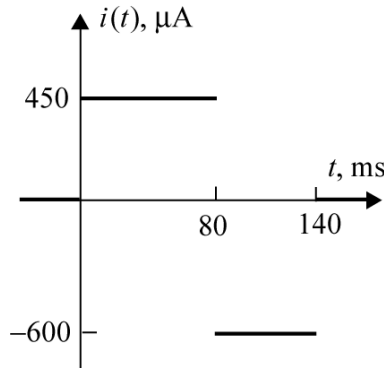


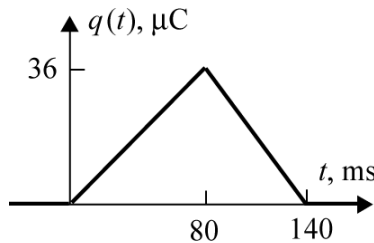
Figure P1.3-5

Solution:

$$q(t) = \int_0^t i(\tau) d\tau = \begin{cases} \int_0^t 450 \mu\text{A} d\tau & \text{when } 0 < t < 80 \text{ ms} \\ (450 \times 10^{-6})(80 \times 10^{-3}) + \int_{80 \text{ ms}}^t (-600 \mu\text{A}) d\tau & \text{when } 80 \text{ ms} < t < 140 \text{ ms} \\ (450 \times 10^{-6})(80 \times 10^{-3}) + (-600 \times 10^{-6})(60 \times 10^{-3}) + \int_{140 \text{ ms}}^t 0 d\tau & \text{when } t > 140 \text{ ms} \end{cases}$$

$$= \begin{cases} (450 \times 10^{-6})t & \text{when } 0 < t < 80 \text{ ms} \\ (36 \times 10^{-6}) + (-600 \times 10^{-6})t & \text{when } 80 \text{ ms} < t < 140 \text{ ms} \\ 0 \text{ C} & \text{when } 140 \text{ ms} < t \end{cases}$$

While $0 < t < 80 \text{ ms}$ $q(t)$ increases linearly from 0 to $36 \mu\text{C}$ and while $80 < t < 140 \text{ ms}$ $q(t)$ decreases linearly from 36 to $0 \mu\text{C}$. Here's the sketch:



P1.3-6 The current in a circuit element is plotted in Figure P1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 μs .

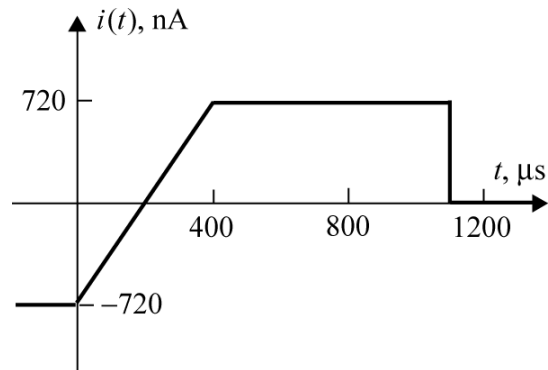
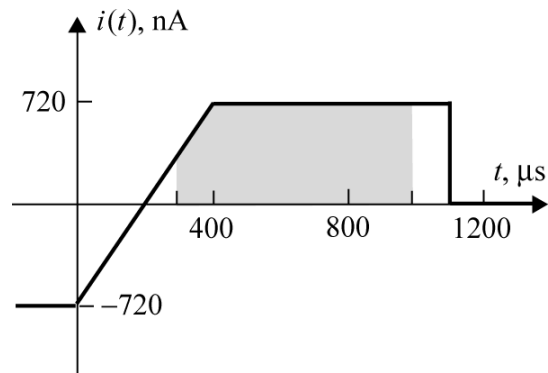


Figure P1.3-6

Solution:

$$q(t) = \int_{300 \mu\text{s}}^{1000 \mu\text{s}} i(\tau) d\tau = \text{"area under the curve between } 300 \mu\text{s and } 1000 \mu\text{s"}$$



$$q(t) = \left(\frac{360 + 720}{2} \times 10^{-9} \right) (100 \times 10^{-6}) + (720 \times 10^{-9}) (600 \times 10^{-6}) = (54 + 432) \times 10^{-12} = 486 \text{ pC}$$

Section 1-5 Power and Energy

P1.5-1 Figure P1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

- (a) Which of the devices supply 30 mW?
- (b) Which of the devices absorb 0.03 W?
- (c) What is the value of the power received by device *B*?
- (d) What is the value of the power delivered by device *B*?
- (e) What is the value of the power delivered by device *C*?

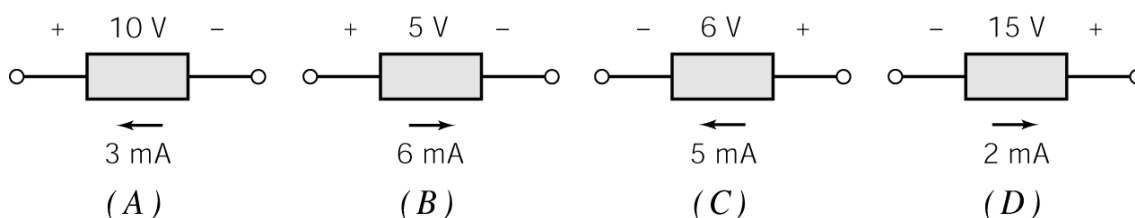


Figure P1.5-1

Solution:

(a) *A* and *D*. The element voltage and current do not adhere to the passive convention in Figures P1.5- *A* and *D* so the product of the element voltage and current is the power supplied by these elements.

(b) *B* and *C*. The element voltage and current adhere to the passive convention in Figures P1.5- *B* and *C* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) 30 mW. The element voltage and current adhere to the passive convention in Figure P1.5- *B*, so the product of the element voltage and current is the power received by this element: $(5 \text{ V})(6 \text{ mA}) = 30 \text{ mW}$. The power supplied by the element is the negative of the power received to the element, -30 W .

(d) -30 mW

(e) -30 mW . The element voltage and current adhere to the passive convention in Figure P1.5- *C*, so the product of the element voltage and current is the power received by this element: $(5 \text{ V})(6 \text{ mA}) = 30 \text{ mW}$. The power supplied by the element is the negative of the power received to the element, -30 W .

P 1.5-2 An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 12 cents per kilowatt-hour, determine the cost of operating the range for two hours.

Solution:

$$\text{a.) } q = \int i dt = i\Delta t = (10 \text{ A})(2 \text{ hrs})(3600 \text{ s/hr}) = \underline{7.2 \times 10^4 \text{ C}}$$

$$\text{b.) } P = vi = (110 \text{ V})(10 \text{ A}) = \underline{1100 \text{ W}}$$

$$\text{c.) } \text{Cost} = \frac{0.12 \$}{\text{kWhr}} \times 1.1 \text{ kW} \times 2 \text{ hrs} = \underline{0.264 \$}$$

P 1.5-3 A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

Solution:

$$P = (6 \text{ V})(10 \text{ mA}) = 0.06 \text{ W}$$

$$\Delta t = \frac{\Delta w}{P} = \frac{200 \text{ W}\cdot\text{s}}{0.06 \text{ W}} = \underline{3.33 \times 10^3 \text{ s}}$$

P 1.5-4 The current through and voltage across an element vary with time as shown in Figure P 1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

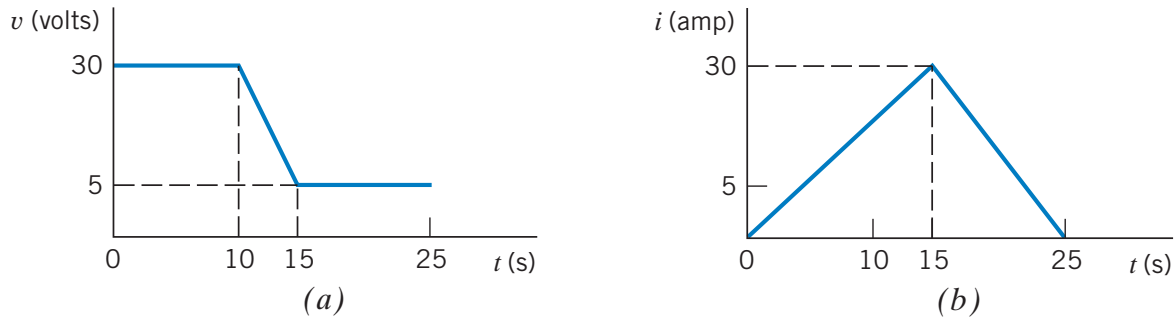


Figure P 1.5-4

Solution:

$$\text{for } 0 \leq t \leq 10 \text{ s: } v = 30 \text{ V and } i = \frac{30}{15}t = 2t \text{ A } \therefore \underline{P = 30(2t) = 60t \text{ W}}$$

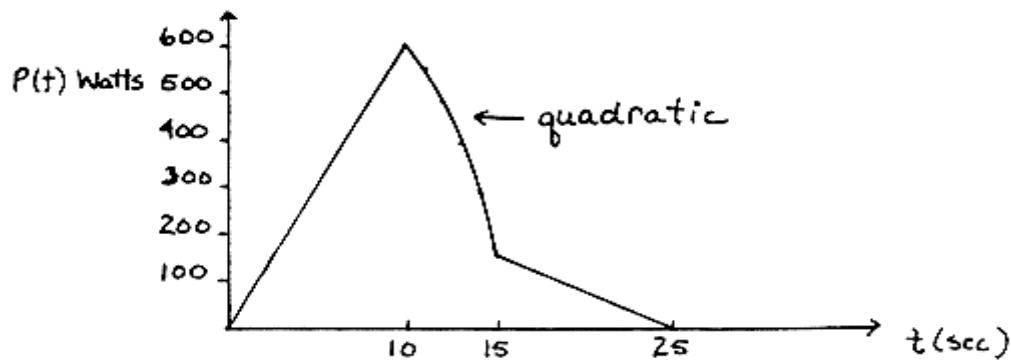
$$\text{for } 10 \leq t \leq 15 \text{ s: } v(t) = -\frac{25}{5}t + b \Rightarrow v(10) = 30 \text{ V } \Rightarrow b = 80 \text{ V}$$

$$v(t) = -5t + 80 \text{ and } i(t) = 2t \text{ A } \Rightarrow \underline{P = (2t)(-5t + 80) = -10t^2 + 160t \text{ W}}$$

$$\text{for } 15 \leq t \leq 25 \text{ s: } v = 5 \text{ V and } i(t) = -\frac{30}{10}t + b \text{ A}$$

$$i(25) = 0 \Rightarrow b = 75 \Rightarrow i(t) = -3t + 75 \text{ A}$$

$$\therefore \underline{P = (5)(-3t + 75) = -15t + 375 \text{ W}}$$



$$\begin{aligned} \text{Energy} &= \int P dt = \int_0^{10} 60t dt + \int_{10}^{15} (160t - 10t^2) dt + \int_{15}^{25} (375 - 15t) dt \\ &= 30t^2 \Big|_0^{10} + 80t^2 - \frac{10}{3}t^3 \Big|_{10}^{15} + 375t - \frac{15}{2}t^2 \Big|_{15}^{25} = \underline{5833.3 \text{ J}} \end{aligned}$$

P 1.5-5 An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where t is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 15 cents/kWh, find the cost of charging the battery for five hours.

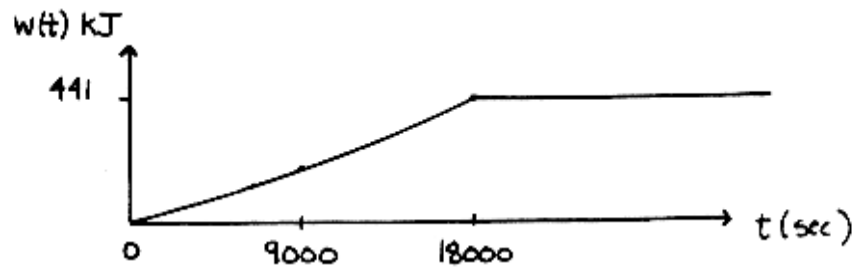
Answer: (b) 1.84 cents

Solution:

a.) Assuming no more energy is delivered to the battery after 5 hours (battery is fully charged).

$$w = \int P dt = \int_0^t vi d\tau = \int_0^{5(3600)} 2 \left(11 + \frac{0.5\tau}{3600} \right) d\tau = 22t + \frac{0.5}{3600} \tau^2 \Big|_0^{5(3600)}$$

$$= 441 \times 10^3 \text{ J} = \underline{441 \text{ kJ}}$$



b.) Cost = $441 \text{ kJ} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{15 \text{¢}}{\text{kWhr}} = \underline{1.84 \text{¢}}$

P 1.5-6 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 4 \sin 3t$ V and $i(t) = (1/12) \sin 3t$ A. Evaluate $p(t)$ at $t=0.5$ s and $t = 1$ s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

Hint: $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

Answer: $p(t) = (1/6)\cos(6t)$ W, $p(0.5) = 0.0235$ W, $p(1) = -0.02466$ W

Solution:

$$p(t) = v(t)i(t) = (4 \cos 3t) \left(\frac{1}{12} \sin 3t \right) = \frac{1}{6}(\sin 0 + \sin 6t) = \frac{1}{6} \sin 6t \quad \text{W}$$

$$p(0.5) = \frac{1}{6} \sin 3 = \underline{0.0235} \quad \text{W}$$

$$p(1) = \frac{1}{6} \sin 6 = \underline{-0.0466} \quad \text{W}$$

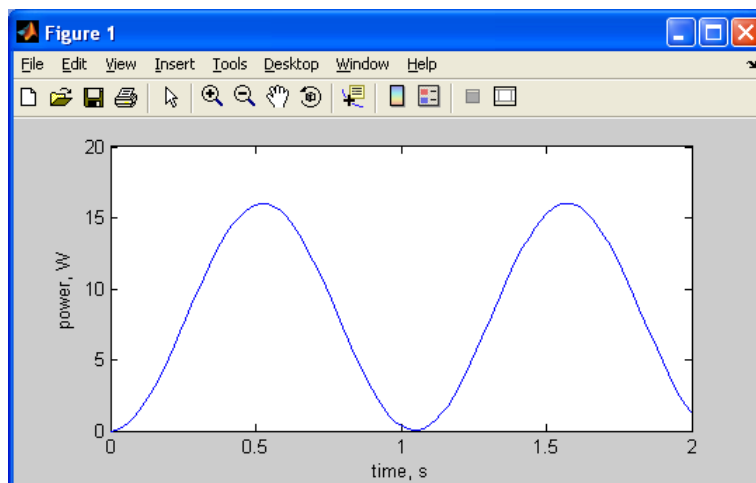
Here is a MATLAB program to plot $p(t)$:

```
clear
t0=0;           % initial time
tf=2;           % final time
dt=0.02;       % time increment
t=t0:dt:tf;    % time

v=4*cos(3*t);  % device voltage
i=(1/12)*sin(3*t); % device current

for k=1:length(t)
    p(k)=v(k)*i(k); % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-7 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 8 \sin 3t$ V and $i(t) = 2 \sin 3t$ A.

Hint: $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

Answer: $p(t) = 8 - 8\cos 6t$ W

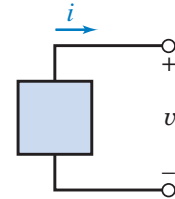


Figure P 1.5-7

Solution:

$$p(t) = v(t)i(t) = (8 \sin 3t)(2 \sin 3t) = 8(\cos 0 - \cos 6t) = \underline{8 - 8 \cos 6t} \text{ W}$$

Here is a MATLAB program to plot $p(t)$:

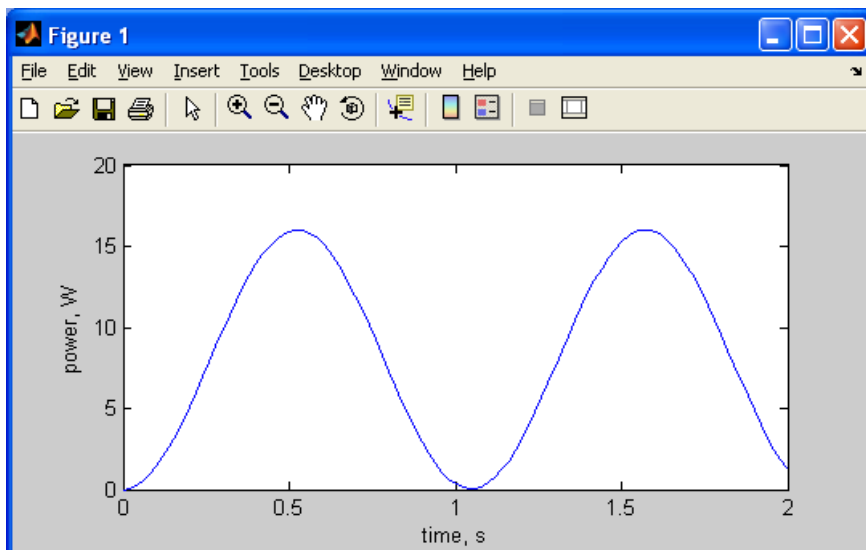
```
clear

t0=0;           % initial time
tf=2;           % final time
dt=0.02;        % time increment
t=t0:dt:tf;     % time

v=8*sin(3*t);   % device voltage
i=2*sin(3*t);   % device current

for k=1:length(t)
    p(k)=v(k)*i(k); % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-8 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6. The element voltage is represented as $v(t) = 4(1 - e^{-2t})$ V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The element current is represented as $i(t) = 2e^{-2t}$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$.

Answer: $p(t) = 8(1 - e^{-2t})e^{-2t}$ W

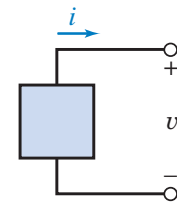


Figure P 1.5-7

Solution:

$$p(t) = v(t)i(t) = 4(1 - e^{-2t}) \times 2e^{-2t} = \underline{8(1 - e^{-2t})e^{-2t}} \text{ W}$$

Here is a MATLAB program to plot $p(t)$:

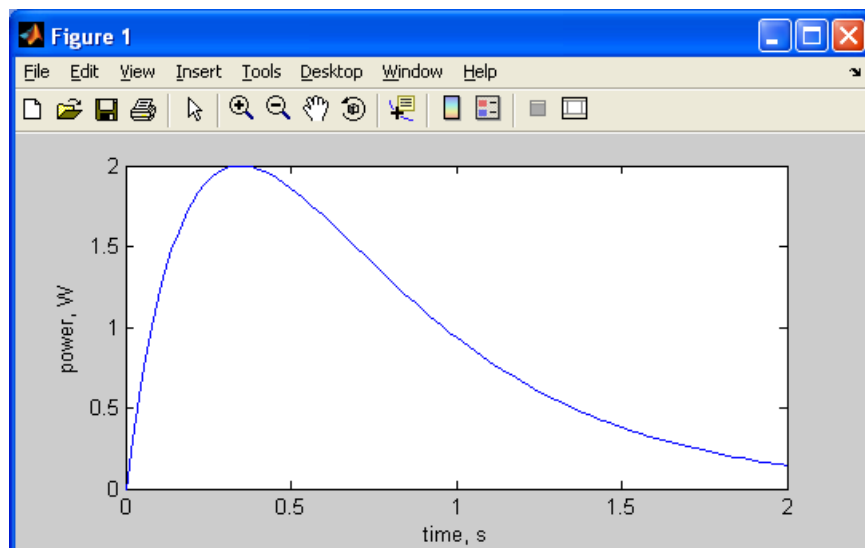
```
clear

t0=0;           % initial time
tf=2;           % final time
dt=0.02;       % time increment
t=t0:dt:tf;    % time

v=4*(1-exp(-2*t)); % device voltage
i=2*exp(-2*t);    % device current

for k=1:length(t)
    p(k)=v(k)*i(k); % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



P 1.5-9 The battery of a flashlight develops 3 V, and the current through the bulb is 200 mA. What power is absorbed by the bulb? Find the energy absorbed by the bulb in a five-minute period.

Solution:

The power is $P = VI = 3 \times 0.015 = \underline{0.045 \text{ W}}$. Next, the energy is $w = P\Delta t = 0.045 \times 5 \times 60 = \underline{13.5 \text{ J}}$.

P1.5-10 Medical researchers studying hypertension often use a technique called “2D gel electrophoresis” to analyze the protein content of a tissue sample. An image of a typical “gel” is shown in Figure 1.5-10a.

The procedure for preparing the gel uses the electric circuit illustrated in Figure 1.5-10b. The sample consists of a gel and a filter paper containing ionized proteins. A voltage source causes a large, constant voltage, 500 V, across the sample. The large, constant voltage moves the ionized proteins from the filter paper to the gel. The current in the sample is given by

$$i(t) = 2 + 30e^{-at} \text{ mA}$$

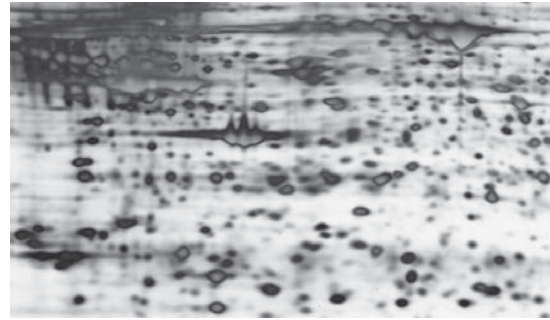
where t is the time elapsed since the beginning of the procedure and the value of the constant a is

$$a = 0.85 \frac{1}{\text{hr}}$$

Determine the energy supplied by the voltage source when the gel preparation procedure lasts 3 hours.

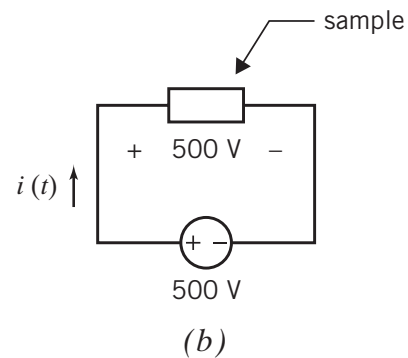
Solution:

$$\begin{aligned} \text{energy} = w(t) &= \int_0^T p(t) dt = \int_0^T v(t)i(t) dt \\ &= \frac{500}{1000} \int_0^3 (2 + 30e^{-0.85t}) dt \\ &= \int_0^3 dt + 15 \int_0^3 (e^{-0.85t}) dt \\ &= (3 - 0) + \frac{15}{-0.85} (e^{-2.55} - 1) \\ &= 3 + 16.3 = 19.3 \text{ J} \end{aligned}$$



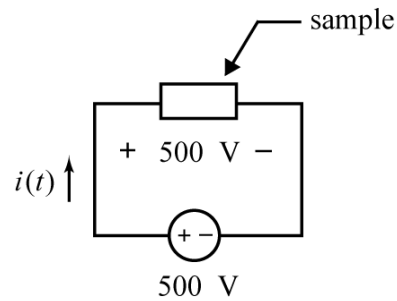
Devon Svoboda, Queen's University

(a)



(b)

Figure 1.5-10 (a) An image of a gel and (b) the electric circuit used to preparation a gel.



Section 1.7 How Can We Check...?

P 1.7-1 Conservation of energy requires that the sum of the power absorbed by all of the elements in a circuit be zero. Figure P 1.7-1 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

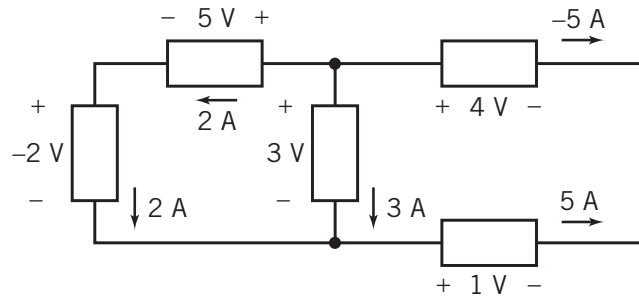


Figure P 1.7-1

Hint: Calculate the power absorbed by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

Solution:

Notice that the element voltage and current of each branch adhere to the passive convention. The sum of the powers absorbed by each branch are:

$$\begin{aligned} (-2 \text{ V})(2 \text{ A}) + (5 \text{ V})(2 \text{ A}) + (3 \text{ V})(3 \text{ A}) + (4 \text{ V})(-5 \text{ A}) + (1 \text{ V})(5 \text{ A}) &= -4 \text{ W} + 10 \text{ W} + 9 \text{ W} - 20 \text{ W} + 5 \text{ W} \\ &= 0 \text{ W} \end{aligned}$$

The element voltages and currents satisfy conservation of energy and may be correct.

P 1.7-2 Conservation of energy requires that the sum of the power absorbed by all of the elements in a circuit be zero. Figure P 1.7-2 shows a circuit. All of the element voltages and currents are specified. Are these voltage and currents correct? Justify your answer.

Hint: Calculate the power absorbed by each element. Add up all of these powers. If the sum is zero, conservation of energy is satisfied and the voltages and currents are probably correct. If the sum is not zero, the element voltages and currents cannot be correct.

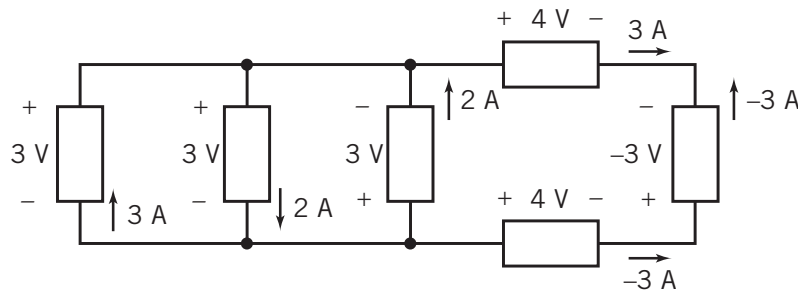


Figure P 1.7-2

Solution:

Notice that the element voltage and current of some branches do not adhere to the passive convention. The sum of the powers absorbed by each branch are:

$$\begin{aligned}
 &-(3\text{ V})(3\text{ A})+(3\text{ V})(2\text{ A})+(3\text{ V})(2\text{ A})+(4\text{ V})(3\text{ A})+(-3\text{ V})(-3\text{ A})+(4\text{ V})(-3\text{ A}) \\
 &= -9\text{ W} + 6\text{ W} + 6\text{ W} + 12\text{ W} + 9\text{ W} - 12\text{ W} \\
 &\neq 0\text{ W}
 \end{aligned}$$

The element voltages and currents do not satisfy conservation of energy and cannot be correct.

P 1.7-3 The element currents and voltages shown in Figure P 1.7-3 are correct with one exception: the reference direction of exactly one of the element currents is reversed. Determine which reference direction has been reversed.

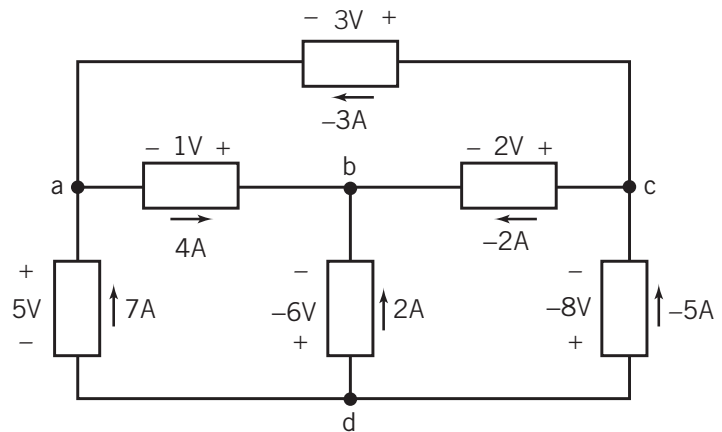


Figure P 1.7-3

Solution:

Let's tabulate the power received by each element. We'll identify each element by its nodes.

nodes	Power received, W
<i>a c</i>	$(3)(-3) = -9$
<i>a b</i>	$-(1)(4) = -4$
<i>b c</i>	$(2)(-2) = -4$
<i>a d</i>	$-(5)(7) = -35$
<i>b d</i>	$(-6)(2) = -12$
<i>c d</i>	$(-8)(-5) = 40$

So

$$\text{Total power received} = -(9 + 4 + 4 + 35 + 12) + 40 = -24 \neq 0$$

Changing the current reference direction for a particular element will change the total power by twice the power of the particular element. Since the element connected between nodes b and d receives -12 W, changing the reference direction of its current will increase the total power received by 24 W, as required. After making that change

$$\text{Total power received} = -(9 + 4 + 4 + 35) + (12 + 40) = 0$$

We conclude that it is the reference direction of the element connected between nodes b and d that has been reversed.

Design Problems

DP 1-1 A particular circuit element is available in three grades. Grade A guarantees that the element can safely absorb $1/2$ W continuously. Similarly, Grade B guarantees that $1/4$ W can be absorbed safely and Grade C guarantees that $1/8$ W can be absorbed safely. As a rule, elements that can safely absorb more power are also more expensive and bulkier.

The voltage across an element is expected to be about 20 V and the current in the element is expected to be about 8 mA. Both estimates are accurate to within 25 percent. The voltage and current reference adhere to the passive convention.

Specify the grade of this element. Safety is the most important consideration, but don't specify an element that is more expensive than necessary.

Solution:

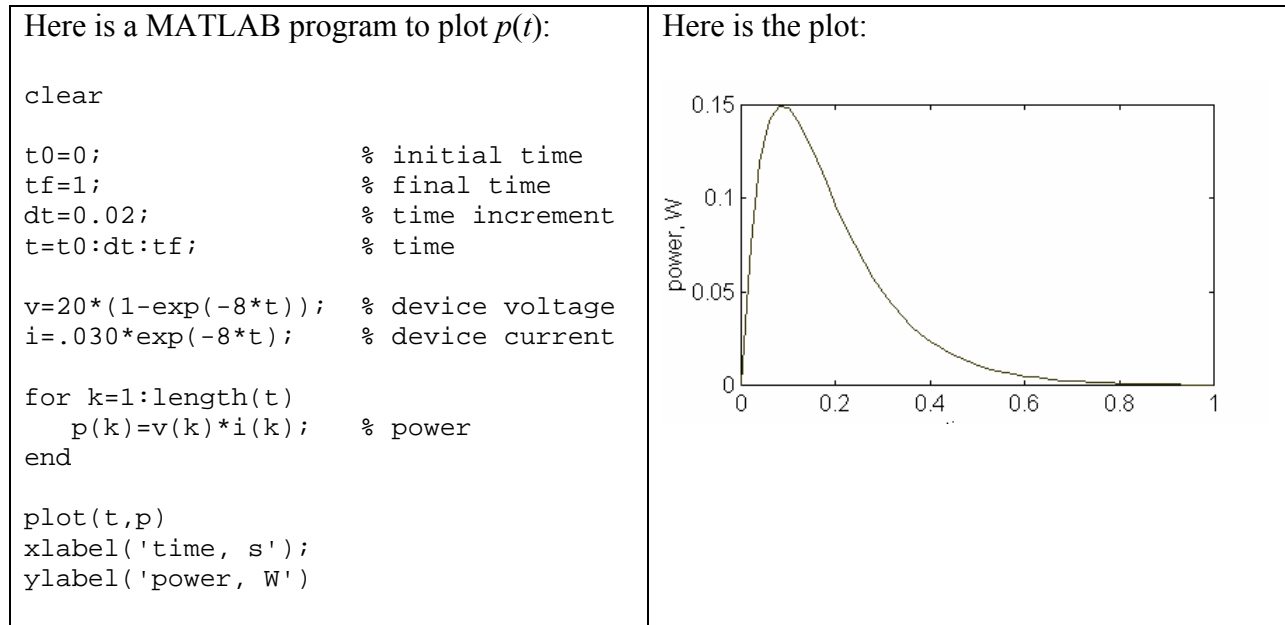
The voltage may be as large as $20(1.25) = 25$ V and the current may be as large as $(0.008)(1.25) = 0.01$ A. The element needs to be able to absorb $(25 \text{ V})(0.01 \text{ A}) = 0.25$ W continuously. A Grade B element is adequate, but without margin for error. Specify a Grade B device if you trust the estimates of the maximum voltage and current and a Grade A device otherwise.

DP 1-2 The voltage across a circuit element is $v(t) = 20(1 - e^{-8t})$ V when $t \geq 0$ and $v(t) = 0$ when $t < 0$. The current in this element is $i(t) = 30e^{-8t}$ mA when $t \geq 0$ and $i(t) = 0$ when $t < 0$. The element current and voltage adhere to the passive convention. Specify the power that this device must be able to absorb safely.

Hint: Use MATLAB, or a similar program, to plot the power.

Solution:

$$p(t) = 20(1 - e^{-8t}) \times 0.03e^{-8t} = \underline{0.6(1 - e^{-8t})e^{-8t} \text{ W}}$$



The circuit element must be able to absorb 0.15 W.